KINEMATIC ANALYSIS AND SYNTHESIS OF CURVED SLOT GENEVA WHEEL

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OCTOBER, 1970

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by

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to the

Department of Mechanical Engineering
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

October, 1970

CERTIFICATE

This is to certify that this work has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENT

The author expresses his debt of gratitude to Dr. J.M. Chakraborty, Assistant Professor, Department of Mechanical Engineering, Indian Institute of Technology, Kanpur for his advice and suggestions in preparing the thesis.

(SURENDRA KUMAR JONEJA)

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LIST OF SYMBOLS

Z	Number of slots
(x_M, y_M)	Coordinates of the point M
R	Radius of the crank
L	Distance between the centre of the Geneva wheel and the centre of rotation of the crank
t	Time
k	Non-dimensional position coefficient for the crank
(X_{12}, Y_{12})	Coordinates of the points of the edges of the slot
$c_0, c_1 .. c_n$	Coefficient of the polynomial
(x, y)	Coordinates of the points on the centre line of the slot
R_1	Radius of the arc I
R_2	Radius of the arc II
c1	Centre of the arc I
c2	Centre of the arc II
(x_{c1}, y_{c1})	Coordinates of the centre of the arc I
(x_{c2}, y_{c2})	Coordinates of the centre of the arc II
(x_p, y_p)	Coordinates of the point of engagement
E	Position of the centre of the roller for $\varphi = 0$
(x_E, y_E)	Coordinates of the point E
m	Given as $\tan(\pi/Z)$
k_ω	First transmission function
k_ϵ	Second transmission function
t_∂	Indexing time

q_ω	Non-dimensional velocity coefficient for the Geneva wheel
q_ψ	Non-dimensional position coefficient for the Geneva wheel
q_ϵ	Non-dimensional acceleration coefficient for the Geneva Wheel
ω_1	Angular velocity of the crank
α_0	Additional working angle
ψ	Angular position of the Geneva wheel
φ	Angular position of the crank
ω	Angular velocity of the Geneva wheel at time t
ϵ	Angular acceleration of the Geneva wheel at time t
λ	Given as $\dfrac{1}{\mathrm{Sin}\,(\pi/Z)}$
φ_0	Angular rotation of the crank for two consecutive stops of the wheel

ABSTRACT

A scheme of computing the coordinates of the slot axis of a Geneva wheel to fit any given acceleration characteristics has been evolved. As the closed form solution becomes a formidable task, the coordinates of the axis of the curved slot are computed in limited number of points covering the whole slot length and then the equation of the axis is found out by fitting the best polynomial through these points by minimising the root mean square error.

To make it possible to cut the curved slot on a conventional milling machine, it is suggested to replace the polynomial by a set of circular arcs. The coordinates of the centres of these circular arcs and their respective radii are found out "optimally" so that the deviation of the generated slot and the desired slot is minimum.

CHAPTER I

INTRODUCTION

1.1 General:

Geneva mechanism has long been popular as a means of producing positive incremental motion. This popularity stems, in part, from the simplicity of mechanism both in design and construction, which makes it comparatively low cost indexing device. In addition, the mechanism inherently produces a precise positioning motion that is necessary for many applications, as for examples, where a simple turret or work table must be indexed, moving a picture projector, used in presses with dial feed and etc.

In the applications where this mechanism is presently utilised, it has proven to be extremely trouble-free and dependable. It is expected that this mechanism may find many applications requiring high speeds. As the speed goes higher, the mechanism, as incremental device, becomes less attractive, because of its kinematic limitations. For instant, a severe limitation under these conditions may result from the high maximum wheel acceleration relative to its average acceleration. This characteristic may cause excessive dynamic loads which, in turn, can cause severe drive pin and slot wear and/or wheel breakage.

Several methods have been employed to reduce the acceleration in order to reduce inertia force and quent wear on the sides of the slot. Among these one is to use non-circular gears attached to driving crank. er one is to place roller on the connecting link of bar linkage. The path of the roller should be curved g the period in which it drives the Geneva wheel.

Therefore the analytical design problem in the of high-speed Geneva mechanism where inertia loads are ant, is one where the best combination design variable ght to reduce inherent kinematic limitations of the nism. Many factors may contribute to a successful a mechanism design, such as material used, surface s, tolerances, loads, stress levels, lubricant etc.

Kinematic synthesis of Geneva wheel contributes iteria of successful design of the mechanism. In t work investigations are done by using the idea of tic synthesis of the mechanism. It is found that jerk nt of enagagement can be reduced appreciably by using crank drive and productivity ratio can be improved t changing number of station points.

iterature Survey :

Most of the work has been done on kinematic is of Geneva wheel. A method presented by Paul H. (1)* makes it possible to calculate the dimensions

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ers in parentheses refer to the references given he end of the thesis.

of all stress members of Geneva drive by establishing maximum angular acceleration of indexed member and thus calculating the value of maximum torque.

Later on more extensive analytical approach to general Geneva mechanism was given by Otto-Litchtwitz (2). He has analysed the motion of regular and irregular Geneva wheel and shown that for given number of stations, the angular motion of Geneva wheel is a function of angular position and angular velocity of driving pin shaft. Angular motion of Geneva wheel is also shown to be independent of its diameter.

In 1956, Ray C. Johnson (3) gave an idea of designing Geneva wheel for minimum contact stress and torsional vibrations. It was seen that mass moment of inertia of Geneva wheel is an important parameter in dynamic analysis. K.Weiss and R.G. Fenton (5) gave a method for finding mass moment of inertia of wheel accurately by using simple equations and chart of coefficients.

C.E. Hasty and J.F. Potts (7) gave general analytical results which can be applied to high-speed Geneva design. Results are derived from classical mechanics theory and provide explicit relationship between performance parameters (those parameters such as contact stress, maximum load etc. which can have significant effect on mechanism performance) and design variables which specify Geneva mechanism (number of slots, wheel diameter, pin diameter etc.). Using these

results, however, it is now possible to synthesize wheel configuration directly.

K.H. Hunt, N. Flink and J. Nayar (4) had already given the idea of getting jerk-free Geneva wheel by having four-bar linkage as driving mechanism. Later on Dr.E.A. Dijksman (6) presented full-length paper on a jerk free motion of an internal Geneva wheel by using special driving mechanism. For this mechanism a four bar is chosen from which the couple point drives the wheel.

Kinematic synthesis of Geneva wheel could not find its popularity with theoreticians. Some work is available on this in Russian and German literatures, but no work seems to have been done in English literature.

1.3 Statement of the problem :

In the present work, by using kinematic synthesis, investigations are carried out with the following objectives:

(1) To reduce jerk at the point of engagement of the Geneva wheel using a simple crank as the driving mechanism.

(2) To improve the productivity ratio which is defined by the ratio of time for indexing (idle time) to the working time (time during which the Geneva wheel is stationary).

(3) By chosing specific angular acceleration curve of Geneva wheel, we can have zero acceleration at the point of engagement, thereby getting the jerk-free Geneva wheel.

(4) By considering zero acceleration for a desired angular rotation of the crank, which leads to a higher productivity ratio.

1.4 Solution Scheme :

For a given angular acceleration curve, the relation between angular position of Geneva wheel and angular position of driving mechanism can be achieved by integrating twice the equation of angular acceleration. Applying the method of transformation of motion, the coordinates of points can be obtained on centre line of slot in terms of angular positions of Geneva wheel and corresponding positions of the crank. The points on the centre line of the slot are approximated by a suitably chosen polynomial. The slot, whose centre line is approximated by the polynomial, is cut by programme-controlled milling machine. In absence of programme-controlled milling machine, it is difficult to cut aforesaid slot. Therefore, the centre line is approximated by optimal set of arcs of circles to get the desired angular acceleration or very close to that.

CHAPTER II

KINEMATIC ANALYSIS OF CURVED-SLOT GENEVA WHEEL

2.1 Basic Characteristic of Geneva Mechanism :

Slots are designed such that crank's roller enters smoothly and for this condition to be satisfied, slot should be radial at the point when crank engages and disengages Geneva wheel.

Slots are also designed in a fashion such that the driven member is fixed not only from the moment of disengagement of the roller with driven mechanism but also during the time when path of the centre of roller is coinciding with the centre line of the curved slot. When the axis of the curved slot is same as the trajectory of the centre of the roller, Geneva wheel will not move and thus it gives additional working angle. This additional working angle is added to working angle already defined in case of radial slot (figure 2.1).This additional working angle is of great use when productivity ratio is required to be improved.

2.2 Kinematic analysis of curved-slot Geneva Wheel :

2.2.1 Sign Conventions used in Kinematic Analysis -

(1) The angle measured in the direction of motion of mechanism is taken to be positive, otherwise negative (figure 2.2).

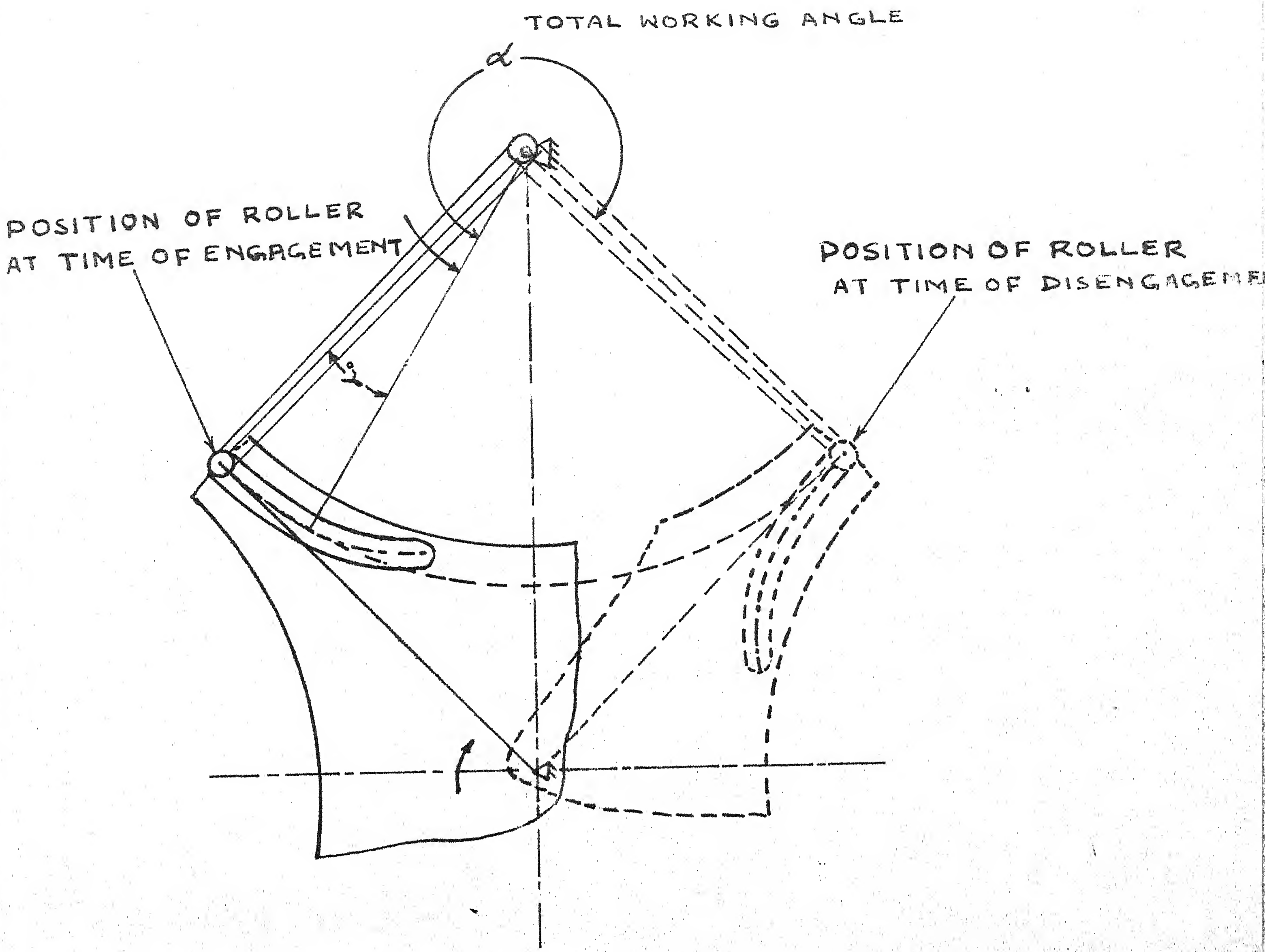

TOTAL WORKING ANGLE
α
POSITION OF ROLLER
AT TIME OF ENGAGEMENT
POSITION OF ROLLER
AT TIME OF DISENGAGEMENT
γ

FIGURE 2.1

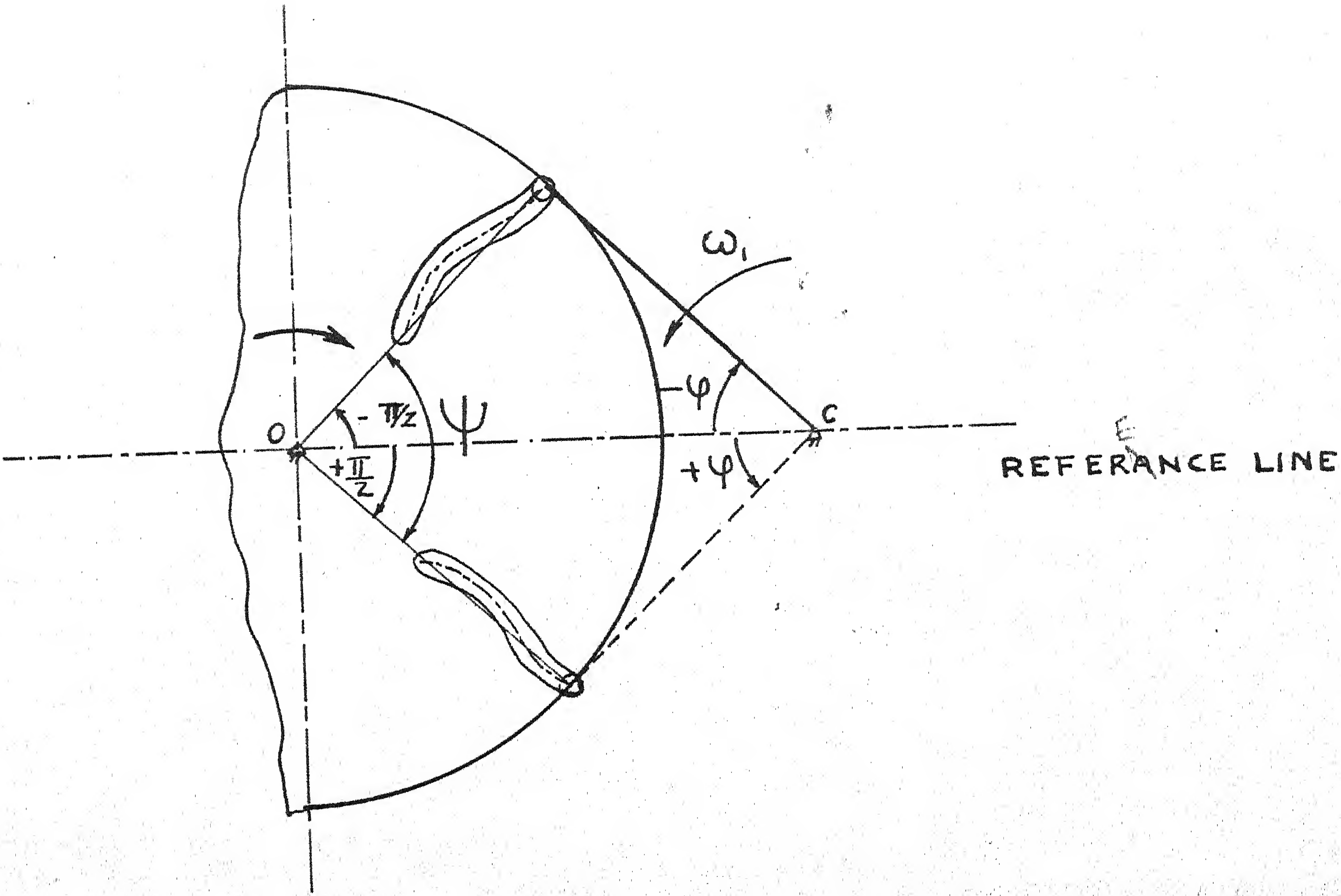


FIGURE 2.2

(2) The reference line for measuring angle is the line joining centre of Geneva wheel and fixed point of the driving crank.

2.2.2 Assumption made in Kinematic Analysis of Wheel -

Driving mechanism is assumed to rotate with a constant angular velocity ω_1.

2.2.3 Some Relevant Definitions -

First Transmission Function :

Angular velocity of Geneva wheel at any time divided by angular velocity of driving crank is defined as first transmission function, denoted by k_ω.

$$k_\omega = (d\psi/dt)/\omega_1 = \frac{d\psi}{d\varphi} \tag{2.1}$$

Second Transmission Function :

Angular acceleration of Geneva wheel at any time divided by square of angular velocity of the driving crank is defined as second transmission function denoted by k_ϵ.

$$k_\epsilon = (d^2\psi/dt^2)/\omega_1^2 = \frac{d^2\psi}{d\varphi^2} \tag{2.2}$$

2.2.4 Kinematic Analysis -

Kinematic characteristics will remain unchanged if the relative position of Geneva wheel and crank is unaltered. At any time the angular position of Geneva wheel and crank are given by ψ and φ respectively (figure 2.3). Referring to figure 2.4, with help of transformation of motion the same relative position can be obtained while Geneva wheel is kept fixed and line OC is rotated in the direction opposite

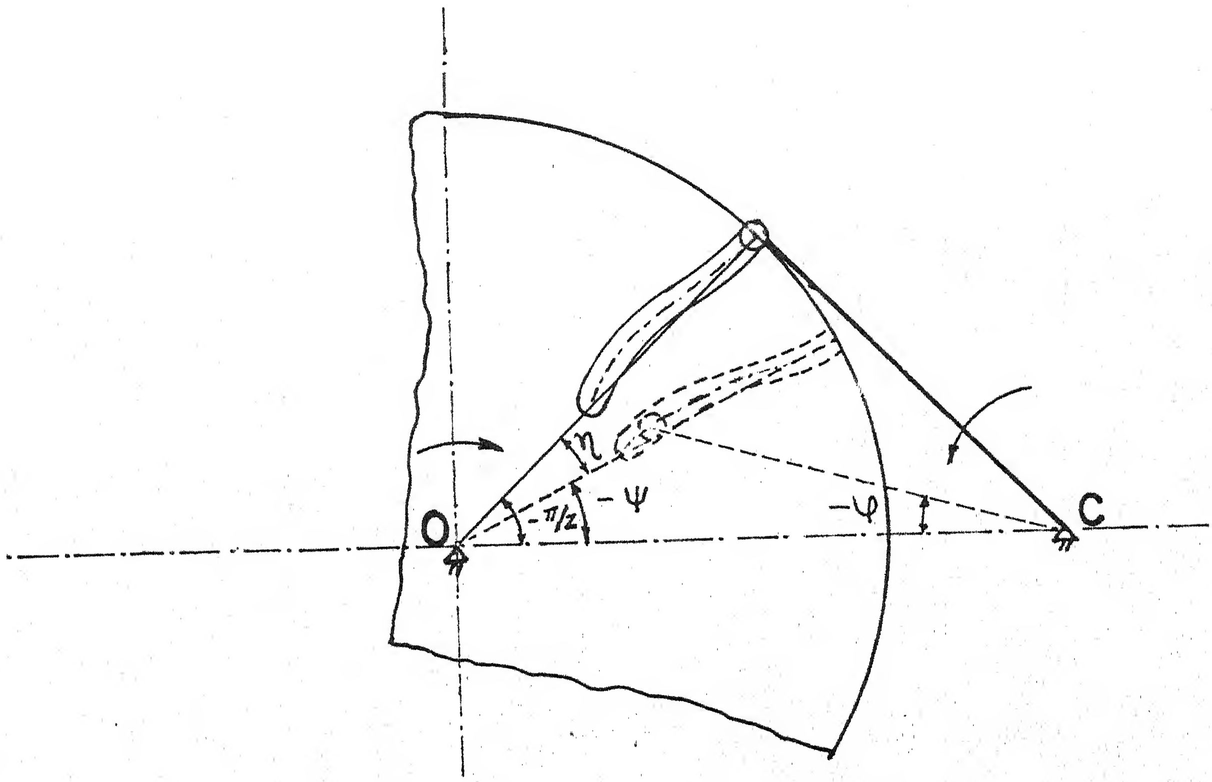


FIGURE 2.3

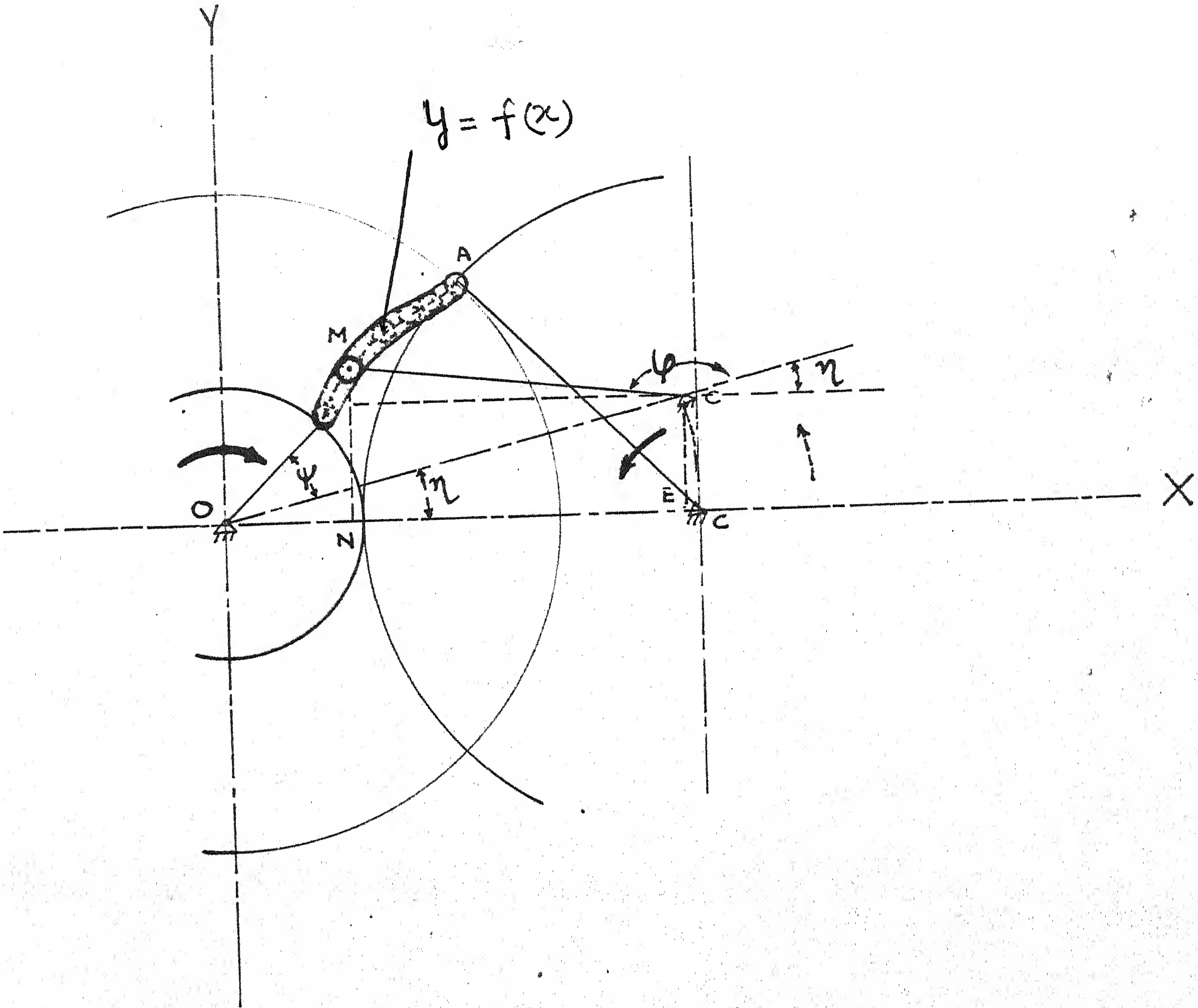


FIGURE 2.4

to the actual motion of Geneva wheel by an angle η , where

$$\eta = \left|-\pi/Z-(-\psi)\right| \text{ such that } 0 \leq \eta \leq \Psi$$

To locate centre of the roller M, an arc of radius equal to the length of crank is taken with centre C' to cut centre line of slot. φ gives the angular position of the crank.

Let the equation of centre line of slot be y = f(x) in x-y coordinate system fixed to Geneva wheel (figure 2.4) Using the aforesaid transformation of motion, the angular position of Geneva wheel and the angular position of the crank can be obtained in terms of known parameters.

The coordinates of point M in figure 2.4 are expressed as follows :

$$x_M = OE-NE = OE - EC'$$
$$= L \cos\eta + R \cos(\eta+\varphi)$$
$$y_M = NB + BM = EC' + BM$$
$$= L \sin\eta + R \sin(\eta+\varphi)$$

or $$x_M = R\left[\lambda \cos\eta + \cos(\eta+\varphi)\right] \quad (2.3)$$

$$y_M = R\left[\lambda \sin\eta + \sin(\eta+\varphi)\right] \quad (2.4)$$

$\lambda = 1/\sin(\pi/z)$

where R - Radius of the crank

L - Distance between the centre of Geneva wheel and the centre of rotation of the crank.

Taking into consideration the sign conventions for angular position of Geneva wheel and angular position of driving mechanism, equations (2.3) and (2.4) can be written as;

$$x_M = R\left[\lambda \cos(\pi/Z+\psi) - \cos(\pi/Z+\psi+\varphi)\right] \quad (2.5)$$

$$y_M = R\left[\lambda \sin(\pi/Z + \psi) - \sin(\pi/Z + \psi + \varphi)\right] \quad (2.6)$$

where Z - Number of slots

General form of equations (2.5) and (2.6) is as follows ;

$$x = R\left[\lambda \cos(\pi/Z + \psi) - \cos(\pi/Z + \psi + \varphi)\right] \quad (2.7)$$

$$y = R\left[\lambda \sin(\pi/Z + \psi) - \sin(\pi/Z + \psi + \varphi)\right] \quad (2.8)$$

From equations (2.7) and (2.8) angular position of the crank and the corresponding position of Geneva wheel can be obtained by the following method ;

Squaring equations(2.7) and (2.8) and adding them we get,

$$x^2 + y^2 = R^2\left[\lambda^2 + 1 - 2\lambda \cos\varphi\right]$$

$$\text{hence } \varphi = \cos^{-1}\left[1/2\lambda\left\{\lambda^2 + 1 - (x^2+y^2)/R^2\right\}\right] \quad (2.9)$$

Equation (2.9) gives the angular position of the driving mechanism for any position of the centre of roller, whose coordinates are known in the $x - y$ system.

Rewriting equation (2.7) in the following form ;

$$x = R\left[\lambda \cos\pi/Z \cdot \cos\psi - \lambda \sin\pi/Z \cdot \sin\psi - \cos(\pi/Z + \varphi)\cdot\cos\psi + \sin(\pi/Z + \varphi)\cdot\sin\psi\right]$$

$$= R\left[\cos\psi\left\{\lambda\cos\pi/Z - \cos(\pi/Z + \varphi)\right\} + \sin\psi(-\lambda\sin\pi/Z + \sin(\pi/Z + \varphi))\right]$$

or

$$x = R\left[a\cos\psi + b\sin\psi\right] \quad (2.10)$$

where $a = \lambda\cos\pi/Z - \cos(\pi/Z + \varphi)$

and $b = \sin(\pi/Z + \varphi) - \lambda\sin\pi/Z$

Similarly equation (2.8) is written as ;

$$y = R\left[\lambda \sin \pi/Z \cdot \cos\psi + \lambda \cos \pi/Z \cdot \sin\psi - (\sin(\pi/Z+\varphi)\cdot \cos\psi + \cos(\pi/Z+\varphi)\cdot \sin\psi)\right]$$

or

$$y = R\left[-b \cos\psi + a \sin\psi\right] \qquad (2.11)$$

From equations (2.10) and (2.11) the value of angular position of Geneva wheel is obtained and is given by the expression ;

$$\psi = \sin^{-1}\left[(bx + ay)/R\cdot(a^2+b^2)\right] \qquad (2.12)$$

First transmission function, a measure of angular velocity of Geneva wheel, is calculated in the following manner.

$$y = R\left[\lambda \sin(\pi/Z+\psi) - \sin(\pi/Z+\psi+\varphi)\right]$$

$$dy/d\varphi = R\left[\lambda \cos(\pi/Z+\psi)\cdot d\psi/d\varphi - \cos(\pi/Z+\psi+\varphi)\cdot(1 + d\psi/d\varphi)\right]$$

or

$$dy/d\varphi = x \cdot d\psi/d\varphi - R \cos(\pi/Z+\psi+\varphi) \qquad (2.13)$$

Similarly from equation (2.8) we can find out -

$$dx/d\varphi = -y \cdot d\psi/d\varphi + R \sin(\pi/Z+\psi+\varphi) \qquad (2.14)$$

In the expression,

$$dy/dx = dy/d\varphi \cdot d\varphi/dx \qquad (2.15)$$

Putting the valuesof $dy/d\varphi$ and $d\varphi/dx$ from equations (2.13) and (2.14) we get;

$$\frac{dy}{dx} = \frac{x\cdot\frac{d\psi}{d\varphi} - R\cos(\pi/Z+\psi+\varphi)}{-y\cdot\frac{d\psi}{d\varphi} + R\sin(\pi/Z+\psi+\varphi)} \qquad (2.16)$$

Thus the first transmission function can be obtained by equations(2.16)and(2.1).

$$\frac{d\psi}{d\varphi} = \frac{R\left[\frac{dy}{dx} \cdot \sin\left(\pi/Z + \psi + \varphi\right) + \cos\left(\pi/Z + \psi + \varphi\right)\right]}{\left(x + y \cdot \frac{dy}{dx}\right)} \qquad (2.17)$$

For calculation of angular acceleration of Geneva wheel, second transmission function can be determined by the following method.

$$\frac{d^2y}{dx^2} = \frac{d}{d\varphi}\left(\frac{dy}{dx}\right) \cdot \frac{d\varphi}{dx}$$

$$= \frac{A + \left(x \cdot \frac{dx}{d\varphi} + y \cdot \frac{dy}{d\varphi}\right) \cdot \frac{d^2\psi}{d\varphi^2} + B}{\left(\frac{dx}{d\varphi}\right)^3} \qquad (2.18)$$

where;

$$A = \frac{d\psi}{d\varphi} \left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2$$

and

$$B = R\left[\sin\left(\pi/Z + \psi + \varphi\right) \cdot \frac{dx}{d\varphi} - \frac{dy}{d\varphi} \cos\left(\pi/Z + \psi + \varphi\right)\right] \cdot \left(1 + \frac{d\psi}{d\varphi}\right)$$

Thus the second transmission function is given by ;

$$\frac{d^2\psi}{d\varphi^2} = \frac{\frac{d^2y}{dx^2}\left(\frac{dx}{d\varphi}\right)^3 - A - B}{x \cdot \frac{dx}{d\varphi} + y \cdot \frac{dy}{d\varphi}} \qquad (2.19)$$

Kinematic characteristics are determined by equations (2.9), (2.12), (2.17) and (2.19) for a given equation of the centre line of the slot in the x - y system fixed to Geneva wheel.

2.3 Concept of Non-dimensional Coefficients :

Number of station points has appreciable effect on the kinematic characteristics of Geneva wheel. To keep uniformity of kinematic characteristics of Geneva wheels

having different number of slots, non-dimensional coefficients are introduced. They may be defined as follows ;

$$q_{\psi} = \psi / \Psi \qquad (2.20)$$

$$q_{\omega} = \omega / (\Psi / t_{\partial}) \qquad (2.21)$$

$$q_{\epsilon} = \epsilon / (\Psi / t_{\partial}^{2}) \qquad (2.22)$$

where Ψ - total angular displacement of Geneva wheel for one complete rotation of crank

ψ - angular position of Geneva wheel at a particular time, t

ω - angular velocity of Geneva wheel at time t

ϵ - angular acceleration of Geneva wheel at time t

t_{∂} - indexing time, i.e. time between two consecutive stops of Geneva wheel

Non-dimensional coefficients are independent of the number of slots but depend upon the law of motion and are variable quantities which can be expressed as function of non-dimensional position - coefficient k, where ;

$$k = \frac{t}{t_{\partial}} = \frac{\varphi}{\varphi_{\partial}} \qquad (2.23)$$

and

t - time

φ - angular position of crank at time t

φ_{∂} - angle of rotation of crank between two consecutive stops of wheel.

Since the crank is rotating with constant angular velocity, the time of indexing is determined by the expression,

$$t_{\gamma} = \frac{\varphi_{\gamma}}{\omega_1} \tag{2.24}$$

2.3.1 Finding Relationship between Transmission Functions and Non-dimensional Coefficients -

From the definitions of the non-dimensional coefficients, we can write the following expressions,

$$\psi = q_{\psi} \Psi \tag{2.25}$$

$$\omega = q_{\omega} \Psi / t_{\gamma} \tag{2.26}$$

$$\epsilon = q_{\epsilon} \Psi / t_{\gamma}^2 \tag{2.27}$$

Putting the value of t_{γ} from equation (2.24) in equations (2.26) and (2.27),

$$\omega = q_{\omega} \cdot \left(\frac{\Psi}{\varphi_{\gamma}} \right) \cdot \omega_1 \tag{2.28}$$

$$\epsilon = q_{\epsilon} \cdot \left(\frac{\Psi}{\varphi_{\gamma}^2} \right) \cdot \omega_1^2 \tag{2.29}$$

From these equations (2.28) and (2.29) first and second transmission functions can be obtained.

First Transmission Function ,

$$k_{\omega} = \frac{d\psi}{d\varphi} = \frac{\omega}{\omega_1} = q_{\omega} \cdot \frac{\Psi}{\varphi_{\gamma}} \tag{2.30}$$

Second Transmission Function,

$$k_{\epsilon} = \frac{d^2\psi}{d\varphi^2} = \frac{\epsilon}{\omega_1^2} = q_{\epsilon} \cdot \frac{\Psi}{\varphi_{\gamma}^2} \tag{2.31}$$

In Geneva wheel slots are equally spaced. Total angle during which Geneva wheel is in contact with the roller of

the crank is given by ψ, where

$$\psi = \frac{2\pi}{Z} \qquad (2.32)$$

And φ_∂ can be calculated from the condition of smooth engagement and disengagement.

Hence

$$\varphi_\partial = \frac{\pi(Z-2)}{Z} \qquad (2.33)$$

If there is additional working angle, φ_∂ is given by

$$\varphi_\partial = \frac{\pi(Z-2)}{Z} - \alpha_o \qquad (2.34)$$

where α_o - additional working angle.

Putting the value of ψ and φ_∂ from equations (2.32) and (2.34) in equations (2.25), (2.30) and (2.31),

$$\psi = f(\varphi) = \frac{2\pi}{Z} \cdot q_\psi \qquad (2.35)$$

$$k_\omega = \frac{d\psi}{d\varphi} = q_\omega \cdot \frac{2\pi}{\pi(Z-2) - Z\alpha_o} \qquad (2.36)$$

$$k_\epsilon = \frac{d^2\psi}{d\varphi^2} = \frac{2\pi Z}{(\pi(Z-2) - Z\alpha_o)^2} \cdot q_\epsilon \qquad (2.37)$$

Equations (2.35), (2.36) and (2.37) give the relations between,

(1) angular position of Geneva wheel and positional non-dimensional coefficient q_ψ

(2) first transmission function and angular velocity non-dimensional coefficient q_ω

(3) second transmission function and angular acceleration non-dimensional coefficient q_ϵ, for Geneva wheel,

respectively.

When $\alpha_o = 0$, expressions for ψ, k_ω and k_ϵ become as follows :

$$\psi = f(\varphi) = \frac{2\pi}{z} q_\psi \qquad (2.38)$$

$$k_\omega = \frac{d\psi}{d\varphi} = q_\omega \cdot \frac{2}{(Z-2)} \qquad (2.39)$$

$$k_\epsilon = \frac{d^2\psi}{d\varphi^2} = \frac{2Z}{\pi(Z-2)^2} \cdot q_\epsilon \qquad (2.40)$$

Angular velocity and angular acceleration of Geneva wheel can be evaluated at any time from the equations (2.28), (2.29) and (2.33) and (2.34), as follows :

$$\omega = \frac{2\pi\omega_1}{\pi(Z-2) - Z\alpha_o} \cdot q_\omega \qquad (2.41)$$

$$\epsilon = \frac{2\pi Z\omega_1^2}{(\pi(Z-2) - Z\alpha_o)^2} \cdot q_\epsilon \qquad (2.42)$$

If $\alpha_o = 0$,

$$\omega = \frac{\omega_1}{(Z-2)} \cdot q_\omega \qquad (2.43)$$

$$\epsilon = \frac{2Z\omega_1^2}{\pi(Z-2)^2} \cdot q_\epsilon \qquad (2.44)$$

From equations (2.38), (2.41) and (2.42) it is evident that when Z, α_o and ω_1 are given, the periodic change in functional position, angular velocity and angular acceleration of Geneva wheel can be obtained by law of change of corresponding non-dimensional coefficient q_ψ, q_ω and q_ϵ .

CHAPTER III

KINEMATIC SYNTHESIS OF GENEVA WHEEL

Kinematic synthesis of Geneva wheel for a desired angular acceleration curve contributes to the criterion of successful design of the mechanism. Following steps may be observed in synthesis of Geneva wheel.

3.1 Finding relation between Angular positions of Geneva wheel and driving crank :

Non-dimensional coefficient for acceleration is a measure of angular acceleration of Geneva wheel. It may be desired to have particular form of acceleration non-dimensional coefficient, which is a function of non-dimensional positional coefficient k for the driving crank. Relation between ψ and φ is obtained by integrating q_ϵ twice.

$$q_\epsilon = F(k) \tag{3.1}$$

$$\begin{aligned} q_\omega &= \int F(k)\, dk \\ &= U(k, c_1) \end{aligned} \tag{3.2}$$

where c_1 - constant of integration

$$\begin{aligned} q_\psi &= \int U(k, c_1)\, dk \\ &= f(k, c_1, c_2) \end{aligned} \tag{3.3}$$

where c_2 - constant of integration.

Constants of integration can be evaluated with the help of boundary conditions at point of engagement i.e.

$q_{\omega} = 0$ and $q_{\psi} = -1/2$ at $k = -1/2$

Equation (3.3) gives relation between q_{ψ} and k, where

$$k = \frac{\varphi}{\varphi_0} \qquad (3.4)$$

and thus from equation (2.35) we can write ;

$$\psi = F(\varphi) = \frac{2\pi}{Z} \cdot q_{\psi} \qquad (3.5)$$

Equation (3.5) gives relation between angular position of Geneva wheel and corresponding position of the driving crank.

3.2 Finding the shape of the slot to suit to the given acceleration curve :

Let φ (i) be angular position of driving crank at any stage 'i' and the corresponding position of Geneva wheel obtained from equation (3.5) is denoted by ψ (i). The equations (2.7) and (2.8) may be written as follows.

$$x\,(i) = R\left[\lambda \cos\left(\pi/Z + \psi\,(i)\right) - \cos\left(\pi/Z + \psi(i) + \varphi(i)\right)\right] \qquad (3.6)$$

$$y\,(i) = R\left[\lambda \sin\left(\pi/Z + \psi(i)\right) - \sin\left(\pi/Z + \psi(i) + \varphi\,(i)\right)\right] \qquad (3.7)$$

Equations (3.6) and (3.7) give the coordinates of point on centre line of slot at i^{th} stage.

Two edges of the slot may be determined with the help of differential calculus, as discussed below. The

equation of circle in the system x - y fixed to wheel with $[x(i), y(i)]$ as centre is given by the following expression;

$$\left[y - y(i)\right]^2 + \left[x - x(i)\right]^2 = r^2 \qquad (3.8)$$

where r - radius of roller

Since x(i) and y(i) are the functions of single variable, envelope of the family of circles given by equation (3.8) can be determined.

$$(y - y(i)) \cdot \left(\frac{dy}{d\varphi}\right)_i + (x - x(i)) \cdot \left(\frac{dx}{d\varphi}\right)_i = 0 \qquad (3.9)$$

where;

$$\left(\frac{dy}{d\varphi}\right)_i = R\left[\lambda \left(\frac{d\psi}{d\varphi}\right)_i \cos\left(\pi/Z + \psi(i)\right) - \left(\left(\frac{d\psi}{d\varphi}\right)_i + 1\right) \cdot \cos\left(\pi/Z + \psi(i) + \varphi(i)\right)\right] \qquad (3.10)$$

$$\left(\frac{dx}{d\varphi}\right)_i = R\left[-\lambda \left(\frac{d\psi}{d\varphi}\right)_i \sin\left(\pi/Z + \psi(i)\right) - \left(\left(\frac{d\psi}{d\varphi}\right)_i + 1\right) \sin\left(\pi/Z + \psi(i) + \varphi(i)\right)\right] \qquad (3.11)$$

Solving equations (3.8) and (3.9), the coordinates of points on two edges of slot are given by ;

$$X_{12} = x(i) \pm \frac{r\left(\frac{dy}{d\varphi}\right)_i}{\left[\left(\frac{dx}{d\varphi}\right)_i^2 + \left(\frac{dy}{d\varphi}\right)_i^2\right]^{1/2}} \qquad (3.12)$$

$$Y_{12} = y(i) \pm \frac{r\left(\frac{dx}{d\varphi}\right)_i}{\left[\left(\frac{dx}{d\varphi}\right)_i^2 + \left(\frac{dy}{d\varphi}\right)_i^2\right]^{1/2}} \qquad (3.13)$$

Subscripts 1 and 2 stand for edge 1 and edge 2, shown in figure 3.1. The upper and lower signs in equations

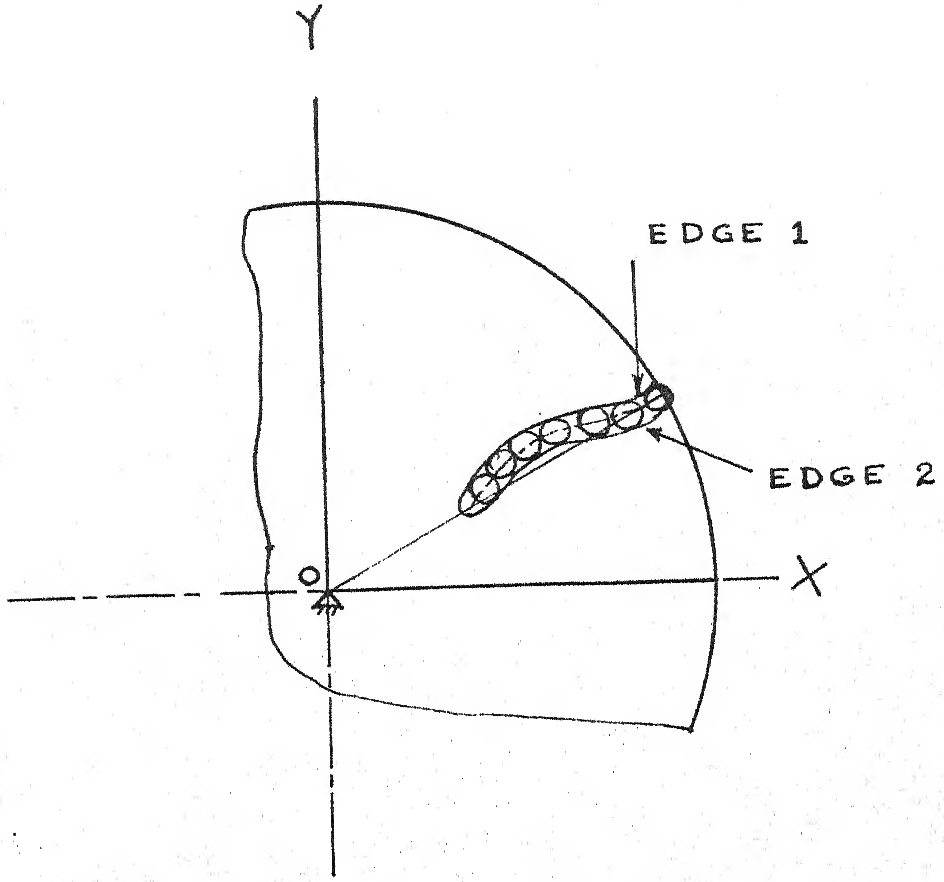


FIGURE 3.1

(3.12) and (3.13) give edge 1 and edge 2 respectively.

It will be a difficult task to get the equation of centre line of the slot in the form $y = f(x)$ by eliminating ψ and φ from equations (3.5), (3.6) and (3.7), since ψ may be an odd function of φ. Therefore from equations (3.5), (3.6) and (3.7) we are able to find discrete points on the centre line of the slot.

3.2.1 Approximating centre line of slot by polynomial -

At time it is desired to have a particular angular acceleration and high productivity ratio. It is difficult and some times almost impossible to have the equation of slot of the form $y = f(x)$, which will give exact angular acceleration curve desired, by eliminating ψ and φ from equations (3.5), (3.6) and (3.7). Therefore centre line of slot is approximated by polynomial which is best fitted through known discrete points $\left[x(i), y(i)\right]$. The slot having polynomial as centre line can be cut by tape-controlled or programme controlled milling machine.

Using 'Least Square Method', suitable polynomial is obtained as follows.

Let the set of points on centre line of the slot be $\left[x(i), y(i)\right]$

where $i = 1,2,\ldots\ldots,N$

Let equation of the polynomial be of the form -

$$Y = c_0 + c_1 x + c_2 x^2 + \ldots\ldots + c_n x^n \tag{3.14}$$

The error in the value of Y calculated at x(i) from the polynomial is Er(i), where,

$$Er(i) = y(i) - (c_o + c_1\, x(i) + c_2\, x(i)^2 \; \ldots\ldots + c_n\, x(i)^n \tag{3.15}$$

The sum of square of the error between Y at x(i) and y(i) is given by H where

$$H = \sum_{i=1}^{N} \left[y(i) - \sum_{J=0}^{n} c_J (x(i))^J \right]^2 \tag{3.16}$$

when c_o, c_1 ....., c_n, values of constant of polynomial are such that the square of the error H is the least, the corresponding equation of polynomial is the desired one. Since the values of x (i) and y(i) are known, therefore partial differentiation with respect to c_J where $J = 0,1,\ldots,n$ will give (n+1) linear equations in c_J.

For minimum value of H the partial differentiation with respect to c_J is zero.

$$\frac{\partial H}{\partial c_k} = 0$$

$$\sum_{i=1}^{N} \left[y(i) - \sum_{J} c_J \quad (x(i))^J \right] . (x(i))^k = 0 \tag{3.17}$$

or

$$\sum_{i=1}^{N} \; y(i) \cdot x(i)^k = \sum_{J}^{n} c_J \sum_{i}^{N} \; (x(i))^{(J+k)}$$

or

$$\rho_k = \sum_{J=0}^{n} \; c_J \; g_{Jk} \tag{3.18}$$

where $k = 0,1,2,\ldots,n$

$$g_{Jk} = \sum_{i} \left[x(i) \right]^{J+k}$$

$$\rho_k = \sum_{i}^{N} x(i)^k \cdot y(i)$$

Expanding equation (3.18)

$$\rho_o = c_o\, g_{oo} + c_1\, g_{10} + c_2\, g_{20} + \ldots + c_n\, g_{no}$$

$$\rho_1 = c_o\, g_{o1} + c_1\, g_{11} + c_2\, g_{21} + \ldots + c_n\, g_{n1}$$

$$\vdots$$

$$\rho_n = c_o\, g_{on} + c_1\, g_{1n} + c_2\, g_{2n} + \ldots + c_n\, g_{nn}$$

Above set of (n+1) linear equations can be solved for c_o, $c_1, \ldots, c_n$ by 'Gauss Jordon' technique.

Knowing the equation of polynomial $Y = f(x)$ for centre line of slot the angular acceleration of Geneva wheel is calculated with the help of the analysis given in Chapter II. There may be some problems in choosing the degree of polynomial. We have started with first degree and gone upto ten degree polynomials. Each one is best fitted through given $\left[x(i), y(i) \right]$. Corresponding to each centre line approximated by polynomial, root mean square value of difference between calculated angular acceleration and desired angular acceleration, is evaluated. The polynomial approximating centre line of slot, which gives the minimum root mean square error of acceleration, is the best to suit the desired goal.

3.2.2 Approximating centre line of slot by arcs of circles-

In absence of programme controlled milling machine, it is very difficult to cut a slot whose centre line is a polynomial. Therefore to cut a slot by ordinary milling machine, the centre line of slot is to be approximated by arcs of circles. If the centre line is replaced by large number of arcs of circles, the results for angular acceleration are closer to the desired angular acceleration of Geneva wheel. But due to practical difficulties, this practice is not adopted. Moreover, at junction of two arcs of circles jerk is observed. Consecutive arcs are always chosen to be tangent to each other at point of junction to have smooth running of the wheel.

Productivity ratio is increased at the cost of high inertia force in radial-slot Geneva wheel. High productivity ratio has great utility in practical field and it can be achieved by approximating the centre line of slot by arcs of circles more effectively than approximating centre line of slot by a polynomial.

Thus two arcs of circles are taken to approximate centre line of slot in the present work. The following method is given for finding optimal set of arcs. On similar line the method for three or more arcs of circles to approximate centre line of slot can be extended.

Optimal Set of Arcs of Circles :

Before finding set of arcs of circles it is advised to save computer time to draw the points $\left[x(i),\ y(i)\right]$ of centre line of slot calculated from equations (3.6) and (3.7). Possible shapes of centre lines of slots are shown in figure 3.2. The plot will give the idea which set of circles should be taken and thus reducing field of search for optimal set of arcs.

Characteristics of Arc I :

From the basic characteristic of Genova wheel at the time of engagement of roller, slot should be radial. Therefore arc I should be tangent to the line AB (figure 3.3). The equation of line AB is given by ;

$$y = mx \tag{3.19}$$

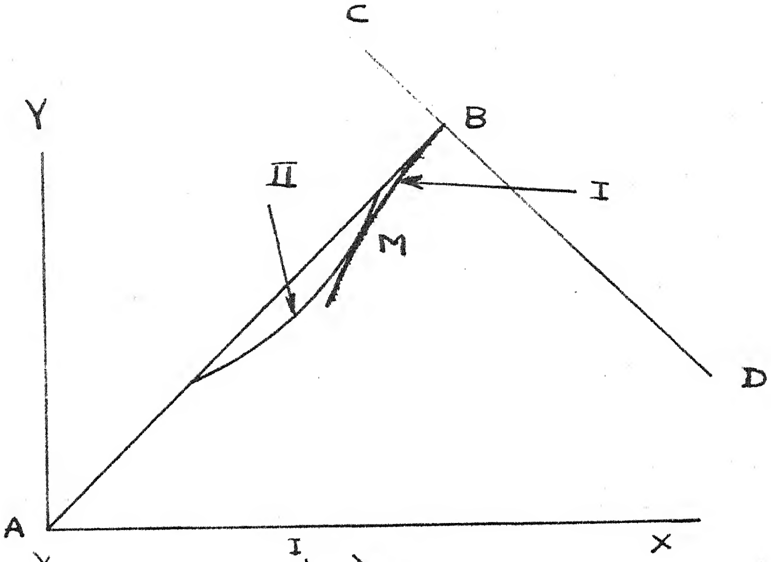
where $m = \tan(\pi/Z)$

Since $y = mx$ is tangent to arc I, the centre of circle I should lie on line CD (figure 3.4), which is perpendicular to line AB. The equation of line CD, which is passing through point of engagement, is given by ;

$$(y-y_p) = (-\frac{1}{m})\,(x-x_p) \tag{3.20}$$

Equation (3.20) gives centres of number of circles which are tangent to line AB. Referring to figure 3.4, let (x_{c1}, y_{c1}) be the coordinates of point c_1 taken on line CD for centre of circle I. Radius of arc I, R_1 is given by;

$$(x_{c1} - x_p)^2 + (y_{c1} - y_p)^2 = R_1^2 \tag{3.21}$$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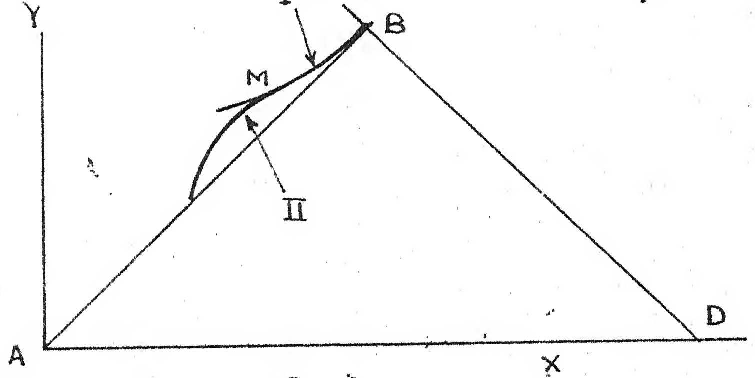
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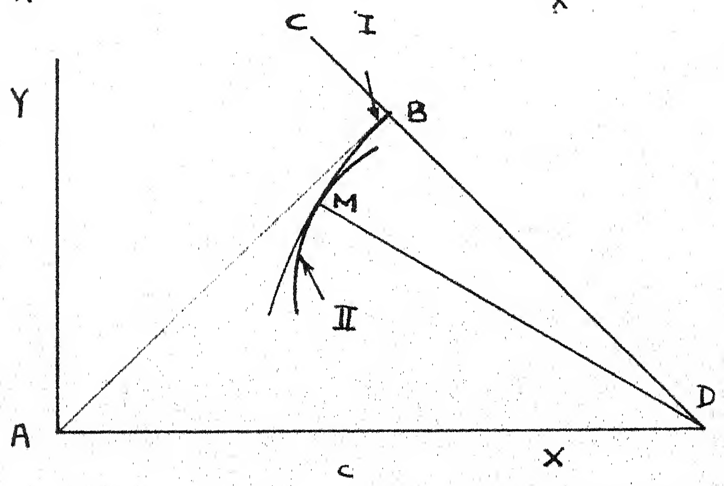
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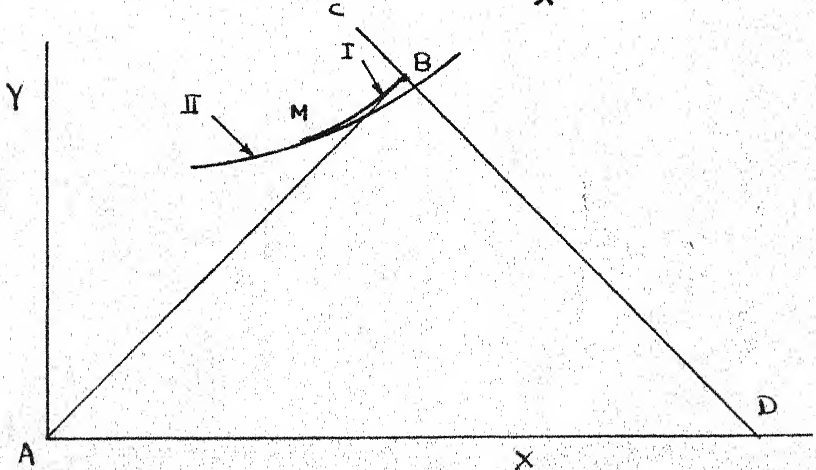
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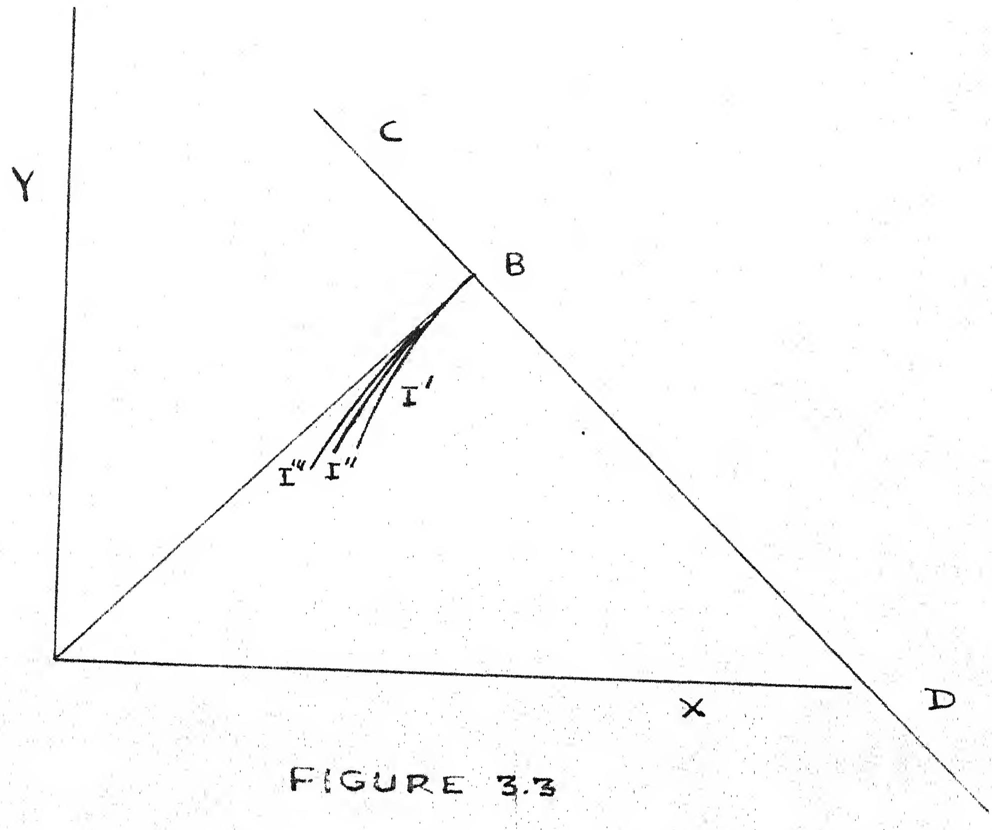


FIGURE 3.3

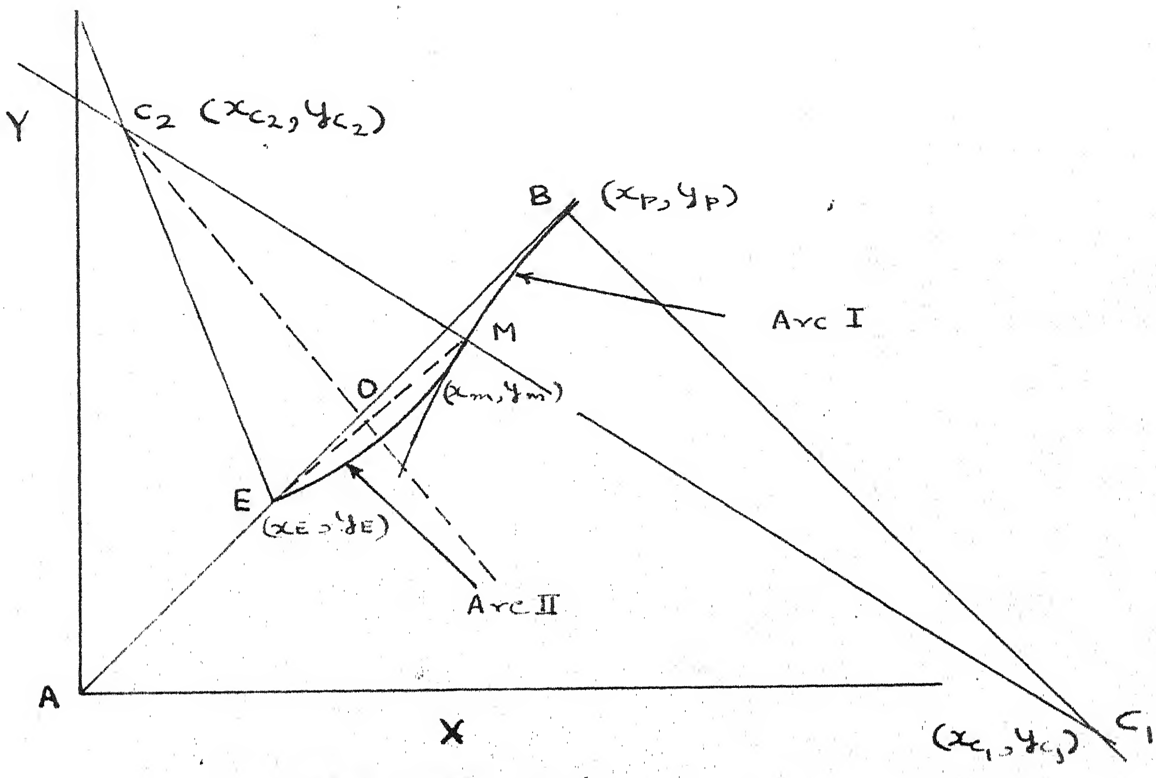


FIGURE 3.4

And equation of arc I is ;

$$(x - x_{c1})^2 + (y - y_{c1})^2 = R_1^2 \qquad (3.22)$$

R_1 is to be determined by search technique as demonstrated later.

Characteristics of Arc II :

(1) Referring to figure 3.4, Arc II should be tangent to arc I at point M which is arbitrarily chosen on arc I such that ;

$x_E \leq x_m \leq x_p$

where;

(x_m, y_m) - coordinates of arbitrary point M which can be exactly determined by search technique and

(x_E, y_E) - coordinates of point E which gives the position of centre of roller at $\varphi = 0$

(2) The centre of the circle for arc II should be on line c_1M so that arc II is tangential to arc I.

(3) The arc II may end at point E or at a point which is in close vicinity of E.

Knowing the coordinates of arbitrary point M, the equation of line c_1M is written as follows;

$$y = \left(\frac{y_{c1} - y_m}{x_{c1} - x_m}\right)x + \left(\frac{x_m\, y_{c1} - y_m\, x_{c1}}{x_m - x_{c1}}\right) \qquad (3.23)$$

Let us assume that arc II ends at point E, whose coordinates are known at $\varphi = 0$. By the given characteristics of arc II, the right bisector of line EM should pass through

point c_2 which gives the centre of arc II (figure 3.4). Let c_2O be right bisector of EM and equation of c_2O is given by;

$$y = -\frac{(x_m - x_E)}{(y_m - y_E)} \cdot x + (y_o + \frac{(x_m - x_E)}{(y_m - y_E)} \cdot x_o) \qquad (3.24)$$

where

(x_o, y_o) - coordinates of point O which is the middle point of EM

Intersection of equations(3.23) and (3.24) gives the centre of arc II and hence the equation of arc II is given by ;

$$(x - x_{c2})^2 + (y - y_{c2})^2 = R_2^2 \qquad (3.25)$$

where

$$R_2^2 = (x_{c2} - x_m)^2 + (y_{c2} - y_m)^2$$

The position of centre of arc II, which is obtained from aforesaid method, is perturbed along line c_1M to get best set corresponding to any fixed arc I.

Arc I is fixed and position of M is changed to get of sets of arcs of circles. Now arc I is changed and same procedure is repeated to get another number of sets of arcs and likewise the optimal set of arcs is to be ascertained. Optimal set of arcs approximating centre line of slot gives the minimum root mean square value of deviation of calculated acceleration curve from the required one.

The mathematical model is difficult to be formulate since there are no constrains on radii of circles.

CHAPTER IV

CALCULATIONS AND RESULTS

To demonstrate the usefulness of the synthesis procedure and authenticity of the derived equations, we take few examples.

4.1 Example 1

To synthesise the Geneva wheel for the angular acceleration curve, which is a combination of a line and a sine curve as given below :

$$q_{\epsilon} = \frac{2A}{1-n}(k + 1/2) \qquad \text{for } -1/2 \leq k \leq -n/2$$

$$q_{\epsilon} = -A \sin \pi k/n \qquad \text{for } -n/2 \leq k \leq 0 \tag{4.1}$$

where n is a constant, such that $.5 \leq n < 1$, and A is a constant for a particular value n.

The kinematic synthesis is done for the Geneva wheel having following specifications.

Z = 6

and L = 12 units.

From the above equation (4.1), the different angular acceleration curves can be got by having different values of n. The example is solved for two values of n which are .5 and .84.

To obtain the relationship between the angular positions of the crank and the Geneva wheel, equation (4.1) is integrated twice. The constants of integrations are evaluated by the boundary conditions, given in Chapter III.

$$q_{\psi} = \frac{A}{3(1-n)} (k + 1/2)^3 - 1/2 \qquad \text{for } -1/2 \leq k \leq -n/2$$

$$q_{\psi} = A\left[\frac{(1-n)}{24} (6k + 2n + 1) + \frac{n^2}{\pi^2} (1 + \sin \pi k/n)\right] - 1/2 \tag{4.2}$$

$$\text{for } -n/2 \leq k \leq 0$$

The value of A can be evaluated by applying the condition that;

$$q_{\psi} = 0 \quad \text{when} \quad k = 0$$

As a result, we get

$$A = \frac{1}{\frac{(1-n)}{12} (1 + 2n) + \frac{2n^2}{\pi^2}} \tag{4.3}$$

From the above relations and the equations obtained from previous Chapters, the following computations are done.

4.1.1 Finding the Points on the Centre line of the Slot :

The set of equations (3.6) and (3.7) are solved for x and y for different crank positions, varying from -60° to 0°. The range is fixed depending on number of slots of the Geneva wheel. The coordinates of different points along with the values of the second transmission function for different positions of the crank, are given in Tables 1 and 2.

4.1.2 Approximating the centre line of the slot by a polynomial :

For the aforesaid sets of points, the following polynomials are chosen to approximate the centre lines from programme 1 givenat the end of the thesis.

for n = .5

$$Y = 4.28769720 - .14272355\ x - .20359804\ x^2 + .04825680\ x^3 - .00251373\ x^4 \qquad (4.4)$$

and for n = .84

$$Y = -36.28260231 + 17.00181627\ x - 2.01881123\ x^2 - 0.06490032\ x^3 + .02680566\ x^4 - .00129695\ x^5 \qquad (4.5)$$

Taking these polynomials as centre lines of the slots, the second transmission functions are calculated for different positions of the driving crank, shown in Tables 3 and 4 and plotted in Figures 4.1a and 4.1b.

4.1.3 Approximating the centre lines by arcs of circles

The method given in the Chapter III is used to obtain the optimal sets of the arcs of circles by using computor. The results are given below.

for n = .5

Arc I

Radius = 15.01

The coordinates of the centre = (16.4, -7.6)

Arc II

Radius = 12.4

The coordinates of the centre = (.88, 14.54)

ANGULAR ACCELERATON CURVES

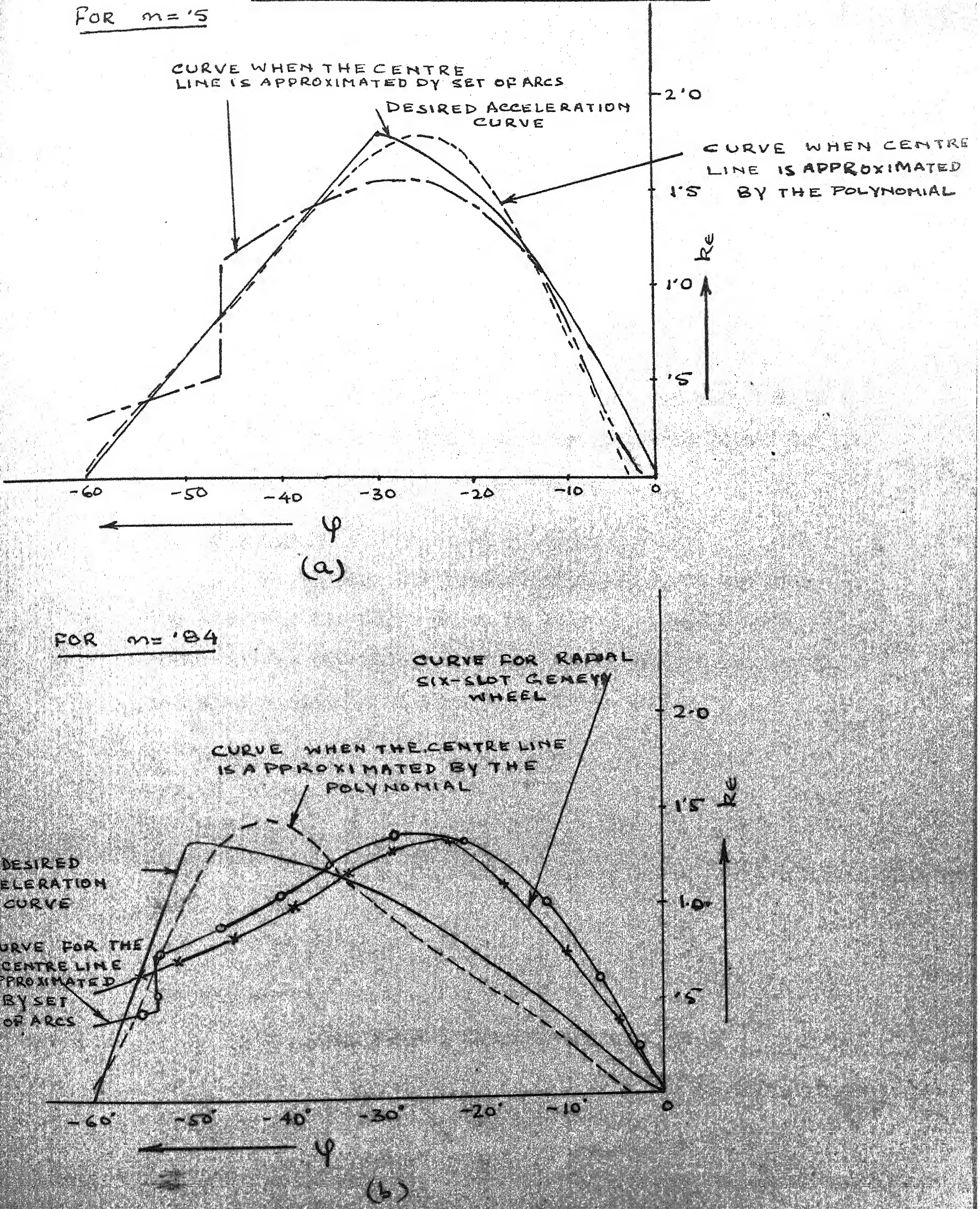


FIGURE 4'1

for n = .84

Arc I

Radius = 20.01

The coordinates of the centre = (19, - 12.12)

Arc II

Radius = 47.5

The coordinates of the centre = (-16.5, 44.8)

With these new centre lines of the slots, the second transmission functions are calculated and are given in Tables 5 and 6. The values of the second transmission functions are plotted in Figures 4.1a and 4.1b.

4.2 Example 2

To improve the productivity ratio, we may have the acceleration curve which is characterised by zero acceleration for the desired angular moment of the crank and then varying with sine law (Figure 4.2).

$$q_{\epsilon} = 0 \qquad \text{for } -1/2 \leq k \leq -1/4$$

$$q_{\epsilon} = A \sin 4\pi k \qquad \text{for } -1/4 \leq k \leq 0 \qquad (4.6)$$

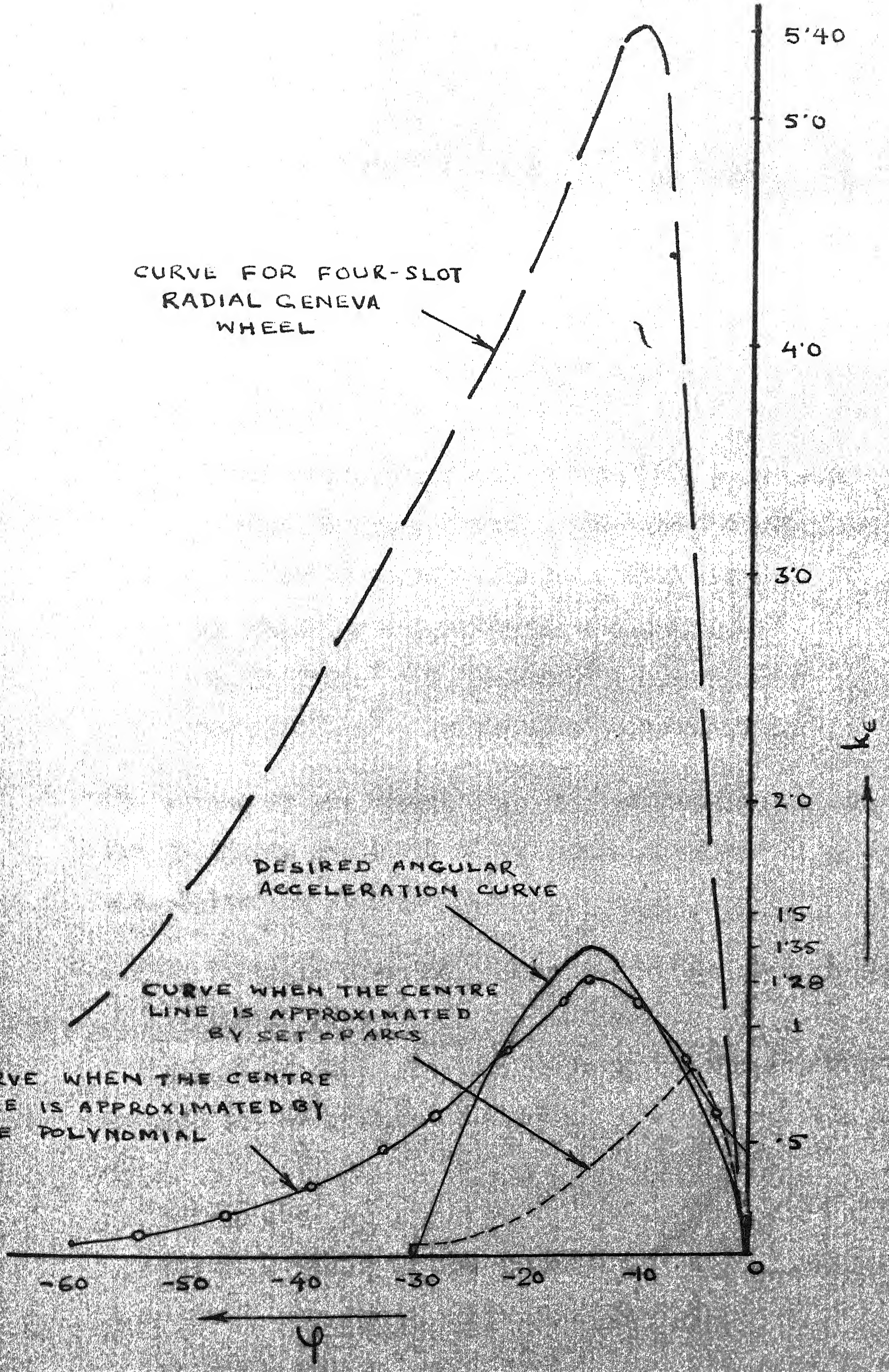
where A = 3.46

given Z = 6

and L = 12 units

The relationship between ψ and φ :

The relationship between the angular positions of the crank and the Geneva wheel is obtained with the help of equations given in the previous Chapters.



ANGULAR ACCELERATION CURVES

FIGURE 4'2

$$\psi = -\pi/Z \qquad \text{for } -1/2 \leq k \leq -1/4$$

$$\psi = \frac{2}{Z}\left[\frac{A}{16}\left(\sin 4\pi k + 4\pi k\right) + \frac{(A-8\quad)}{16}\right] \qquad (4.7)$$

$$\text{for } -1/4 \leq k \leq 0$$

4.2.2 Finding the points on the centre line of the slot

The values of x and y are calculated and are shown in Table 7.

4.2.3 Approximating the centre line by a polynomial

For aforesaid set of points the following polynomial is chosen to approximate the centre line.

$$\begin{aligned} Y = & -68.78252125 + 1.04311174x + 8.28429866x^2 \\ & -2.05107731x^3 + 0.27392938x^4 - 0.02928159x^5 \\ & +0.00198395x^6 - 0.00002857x^7 - 0.00000226x^8 \qquad (4.8) \end{aligned}$$

The values of the second transmission function calculated for different positions of the crank are given in Table 8 and plotted in Figure 4.2.

4.2.4 Approximating the centre line of the slot by the set of arcs :

Using the procedure given in the Chapter III, the optimal set of arcs is obtained.

<u>Arc I</u>

Radius = 6

The coordinates of the centre = (12, 0)

<u>Arc II</u>

Radius = 6.35

The coordinates of the centre = (12.3, -.17)

With this new centre line of the slot, the values of the second transmission function are calculated and given in Table 9. Figure 4.2 gives the comparative picture of the angular acceleration curves which are obtained when ;

(1) the centre line is approximated by a polynomial

and (2) the centre line is approximated by the set of arcs.

TABLE 1

The calculated values of x, y and desired k_ϵ

For n = .5

φ	ψ	x	y	k_ϵ
-60.0	-30.000	9.000	5.196	0
-54.0	-29.963	8.470	4.860	.356
-48.0	-29.701	7.962	4.500	.713
-42.0	-28.982	7.469	4.147	1.069
-36.0	-27.868	7.087	3.883	1.354
-30.0	-25.336	6.537	3.543	1.782
-24.0	-21.983	6.114	3.326	1.694
-18.0	-17.572	5.747	3.165	1.441
-12.0	-12.264	5.460	3.056	1.047
- 6.0	- 6.304	5.272	2.999	.551
0	0	5.196	3.000	0

TABLE 2

The calculated values of x, y and desired k_ϵ

For n = .84

φ	Ψ	x	y	k_ϵ
-60.0	-30.000	9.000	5.196	0
-55.2	-29.966	8.572	4.934	.668
-50.4	-29.641	8.146	4.674	1.336
-45.6	-28.837	7.713	4.444	1.320
-40.8	-27.501	7.280	4.241	1.276
-36.0	-25.653	6.856	4.058	1.200
-32.2	-23.322	6.460	3.886	1.104
-26.4	-20.548	6.098	3.720	.979
-21.6	-17.382	5.784	3.558	.832
-16.8	-13.880	5.529	3.403	.668
-12.0	-10.112	5.341	3.259	.488
- 7.2	- 6.147	5.227	3.128	.297
- 2.4	- 2.060	5.188	3.035	.100
0	0	5.196	3.000	0

TABLE 3

Results obtained when the centre line of the slot is approximated by the polynomial

For n = .5

x	y	φ	ψ	k_ϵ
9.0	5.20	-60.010	-29.989	.029
8.6	4.95	-55.492	-29.987	.306
8.2	4.67	-50.829	-29.875	.568
7.8	4.38	-46.047	-29.532	.836
7.4	4.100	-41.142	-28.834	1.115
7.0	3.829	-36.108	-27.627	1.396
6.6	3.580	-30.857	-25.712	1.642
6.2	3.363	-25.242	-22.793	1.772
5.8	3.182	-22.202	-22.809	1.750
5.4	3.041	-10.497	-10.770	.810
5.2	3.000	- 3.000	- 4.2	0

TABLE 4

Results obtained when the centre line of the slot is approximated by the polynomial

For n = .84

x	y	φ	ψ	$k\epsilon$
9.00	5.191	-59.960	-30.040	.086
8.6	4.959	-55.553	-29.92	.432
8.2	4.715	-51.046	-29.660	.967
7.8	4.484	-46.541	-29.245	1.354
7.4	4.283	-40.954	-27.635	1.452
7.0	4.110	-37.589	-26.380	1.342
6.6	3.952	-32.963	-24.196	1.108
6.2	3.785	-27.925	-21.352	.687
5.8	3.510	-20.146	-16.357	.582
5.4	3.278	-12.980	-11.430	.412
5.2	3.008	- 4.01	- 3.91	.001

TABLE 5

Results obtained when the centre line is approximated by the set of arcs.

For n = .5

x	y	φ	ψ	k_ϵ	
9.0	5.196	-60.000	-30.000	.343	
8.6	4.957	-55.440	-29.938	.395	Arc I
8.2	4.700	-50.974	-29.731	.460	
7.8	4.494	-46.244	-29.340	.544	
7.8	4.494	-46.244	-29.340	1.108	
7.4	4.150	-41.399	-28.599	1.254	
7.0	3.898	-36.466	-27.318	1.393	
6.6	3.667	-31.335	-25.357	1.507	Arc II
6.2	3.454	-25.812	-22.477	1.552	
5.8	3.261	-19.513	-18.188	1.434	
5.4	3.063	-11.049	-10.927	.887	
5.2	3.002	- 1.497	- 1.502	.127	

TABLE 6

Results obtained when the centre line is approximated by the set of arcs

For n = .84

x	y	φ	ψ	k_e	
9.0	5.196	-60.000	-30.000	.404	Arc I
8.4	4.836	-53.300	-29.830	.499	
8.4	4.836	-53.300	-29.830	.758	
7.8	4.467	-46.453	-29.136	.926	
7.2	4.110	-39.402	-27.631	1.115	
6.6	3.764	-31.886	-24.956	1.302	Arc II
6.0	3.430	-23.323	-20.346	1.395	
5.4	3.107	-11.348	-11.006	.982	
5.2	3.002	- 1.641	- 1.642	.079	

TABLE 7

The calculated values of x, y and desired k_ϵ

φ	ψ	x	y	k_ϵ
-60.0	-30.000	9.000	5.196	0
-54.0	-30.000	8.473	4.864	0
-48.0	-30.000	7.985	4.459	0
-42.0	-30.000	7.541	4.014	0
-36.0	-30.000	7.146	3.527	0
-30.0	-30.000	6.804	3.000	0
-24.0	-29.951	6.517	2.446	.792
-18.0	-29.631	6.282	1.895	1.284
-12.0	-28.871	6.105	1.368	1.284
- 6.0	-27.678	6.002	.872	.792
- 0	-26.203	5.987	.307	0

TABLE 8

Results obtained when the centre line is approximated by the polynomial

x	y	φ	ψ	k_ϵ
9.0	5.21	-60.02	-29.98	.095
8.6	4.95	-52.38	-29.94	.100
8.2	4.65	-46.52	-29.85	.150
7.8	4.35	-38.38	-29.66	.380
7.4	3.62	-32.32	-29.46	.480
7.0	3.22	-22.20	-29.41	.980
6.6	2.44	-15.35	-28.55	1.280
6.2	1.87	- 6.25	-27.75	1.010
5.9	.69	- 0	-26.51	.460

TABLE 9

Results obtained when the centre line is approximated by the set of arcs

x	y	φ	ψ	k_e	
9.00	5.196	-60.000	-30.000	0	
8.60	4.852	-53.561	-30.000	0	
8.20	4.661	-49.672	-30.000	0	
7.80	4.262	-43.573	-30.000	0	Arc I
7.60	4.113	-42.567	-30.000	0	
7.20	3.636	-36.872	-30.000	0	
6.80	3.001	-30.000	-30.000	0	
6.80	3.001	-30.000	-30.000	.053	
6.40	2.174	-21.187	-29.865	.137	Arc II
6.00	.614	- 4.395	-28.538	.782	
5.80	.387	- .002	-26.215	.403	

CHAPTER V

DISCUSSION

5.1 Discussion of the Results :

The plots indicate the difference between the calculated angular accelerations and desired accelerations. At time we need to synthesise for jerk free Geneva wheel. Angular acceleration curve, characterised by zero acceleration at the time of engagement of Geneva wheel, is taken. It is found in example 1, that the centre line of the slot approximated by the polynomial gives very low jerk at the time of engagement of wheel with roller. Approximating the centre line by arcs of circles gives less jerk in comparison with radial slot Geneva wheel having the same number of slots. But at the junctions of two consecutive arcs jerk is observed - its value depends on the value of $\frac{d^2y}{dx^2}$ at the junction point.

For improving productivity ratio, Geneva wheel is synthesized for the angular acceleration curve which is characterised by zero acceleration for an angle of rotation of the crank which is equal to the additional working angle (Example 2).

The Figure 4.2 shows that the better results to improve productivity ratio are obtained when centre line is approximated by the optimal set of arcs instead of by the polynomial. The productivity ratio obtained in Example 2 can

also be obtained by using a Geneva wheel having four radial slots. The radial slot Geneva wheel has greater value of maximum angular acceleration for the same productivity ratio (Figure 4.2).

At the time when we desire to have a particular angular acceleration curve, we may approximate the centre line according to manufacturing facilities available.

5.2 Conclusions :

The following conclusions are drawn from the results obtained.

1. For the synthesis of a jerk free Geneva wheel the centre line of the slot should be approximated by a polynomial.
2. For the synthesis of a Geneva wheel to improve productivity ratio the centre line is approximated by the optimal set of arcs.

5.3 Scope of further work :

There are many features in the field of kinematic analysis and synthesis of Geneva wheel which need more rigorous mathematical approach. Some of them are listed below.

(a) Effect of allowance between the roller and the slot on the kinematic characteristics of Geneva wheel.

(b) Synthesis of Geneva wheel with varying slot width to have independent acceleration and

decelloration characteristics.

(c) Design of Geneva wheel to give minimal dynamic load on the system.

APPENDIX

COMPUTER PROGRAMME

<u>Programme 1</u> - is used to obtain the equation of best suited polynomial given by equation (3.14) for the calculated values of x and y.

<u>Programme 2</u> - is used to get the optimal set of the circular arcs given by equations (3.22) and (3.25) to approximate the centre line of the slot.

```
      DIMENSION X(2000),Y(2000),YPI(2000)
      DIMENSION C(21)
      PI=4.*ATAN(1.)
      READ11,Z,E
      FORMAT(2F8.2)
      Q=1./SIN(PI/Z)
      CL=12.
      RC IS RADIUS OF THE CRANK
      RC=CL*SIN(PI/Z)
      RA=(2.*E*E)/(PI*PI)+(1.-E)*(1.+2.*E)/12.
      A=1./RA
      DO 7 I=1,26
      V=I-1
      D=.02*V
      S=-.5+D
      FIR=(PI*(Z-2.)/Z)*S
      FI IS THE ANGULAR POSITION OF THE CRANK
      FI=180.*FIR/PI
       IF(I.GT.2) GO TO 12
      SAE THE ANGULAR POSITION OF THE GENEVA WHEEL
      SAE=(2.*PI*A*(S+.5)**3)/(Z*3.*(1.-E))-PI/Z
      ACCN=(4.*Z*A*(S+.5))/(PI*(1.-E)*(Z-2.)**2)
      GO TO 9
      B=(1.-E)*(6.*S+2.*E+1.)/24.
      TA=(E*E*(1.+SIN(PI*S/E)))/(PI*PI)
      D=2.*PI*A/Z
      SAE=D*(B+TA)-PI/Z
      ACCN IS THE SECOND TRANSMISSION FUNCTION
      ACCN=-(2.*Z*A*SIN(PI*S/E))/(PI*(Z-2.)**2)
      SAED=180.*SAE/PI
      W=PI/Z+SAE
      X(I) AND Y(I) ARE THE SET OF PIVOTED POINTS
      Y(I)=(Q*SIN(W)-SIN(W+FIR))*RC
      X(I)=(Q*COS(W)-COS(W+FIR))*RC
      PRINT3,FI,SAED,ACCN,X(I),Y(I)
 3    FORMAT(10X,5F16.6)
 7    CONTINUE
      DO 40 N=3,10
      J=26
      K=N
      YPI  IS THE VALE OF Y CALCULATED FROM THE POLYNOMIAL
      CALL FITWEL(X,Y,N,J,C,ER,YPI,RMS )
      PRINT 24,(C(I),I=1,K)
 24   FORMAT(5X,10F12.8//)
      PRINT25,ER,RMS
 25   FORMAT(//10X,2F16.6///)
 40   CONTINUE
      STOP
      END
$IBFTC FITWEL
      SUBROUTINE FITWEL(X, Y, M, NUMBER, C, ER, YP, RMS)
C     THIS PROGRAMME IS USED TO FIND THE COEFFICIENTS  OF  M ORDER EQUATION
C     WHICH REPRESENTS THE BEST CURVE FOR GIVEN POINTS   ORDER SHOULD NOT
```

```
C       ECEED 10  AND NUMBER OF  POINTS SHOULD NOT BE MORE THAN 200
        DIMENSION X(2000), Y(2000)
        DIMENSION YP(2000)
        DIMENSION C(21), A(21, 21),B(21), P(21)
        MX2 = M * 2
        DO 13 I = 1, MX2
        P(I) = 0.0
        DO 13 J = 1, NUMBER
13      P(I) = P(I) + X(J)**I
        N = M + 1
        DO 30 I = 1, N
        DO 30 J=1,N
        K = I + J - 2
        IF (K) 29,29,28
28      A(I, J) = P(K)
        GO TO 30
29      A(1, 1) = NUMBER
30      CONTINUE
        B(1) = 0.0
        DO 21 J = 1, NUMBER
21      B(1) = B(1) + Y(J)
        DO 22 I = 2, N
        B(I) = 0.0
        DO 22 J = 1, NUMBER
22      B(I) = B(I) + Y(J) * X(J)**(I -1)
        NM1 = N-1
        DO 300 K = 1, NM1
        KP1 = K+ 1
        L = K
        DO 400 I = KP1, N
        IF ( ABS(A(I, K)) - ABS(A(L, K)))  400, 400, 401
401     L = I
400     CONTINUE
        IF (L-K) 500, 500, 405
405     DO 410 J = K, N
        TEMP = A(K, J)
        A(K, J) = A(L, J)
410     A(L, J) = TEMP
        TEMP = B(K)
        B(K) = B(L)
        B(L) = TEMP
500     DO  300 I = KP1, N
        FACTOR = A(I, K) / A(K, K)
        A(I, K) = 0.0
        DO 301 J = KP1, N
301     A(I, J) = A(I, J) - FACTOR * A(K, J)
300     B(I) = B(I) - FACTOR * B(K)
        C(N) = B(N) / A(N, N)
        I = NM1
710     IP1 = I + 1
        SUM = 0.0
        DO 700 J = IP1, N
700     SUM = SUM + A(I, J) * C(J)
C       C(I) ARE THE CONSTANTSOF THE POLYNOMIAL
```

```
      C(I) = (B(I) - SUM) / A(I, I)
      IF (I) 800, 800, 710
800   CONTINUE
      CALL CHECK(X,Y,M,C,NUMBER,RMS,YP,ER)
      RETURN
      END
$IBFTC CHECK
      SUBROUTINE CHECK(X,Y,M,C,NUMBER,RMS,YP,ER)
      DIMENSION X(2000), Y(2000)
      DIMENSION C(21)
      DIMENSION YP(2000)
      MP1 = M + 1
      ER = 0.
      RMS = 0.0
      DO  32  I = 1, NUMBER
      YP(I) = 0.
      DO 31  J = 1, MP1
      K = J - 1
      IF ( K . NE . 0 )    GO TO 30
      YP(I) = C(J)
      GO TO 31
   30 YP(I) = YP(I) + C(J) * X(I) ** K
   31 CONTINUE
      EH = ABS( Y(I) - YP(I))
      RMS = RMS + EH * EH
      IF(EH.GT.ER)  ER=EH
   32 CONTINUE
      RMS = SQRT(RMS)
      XNU=NUMBER
      RMS=RMS/XNU
      RETURN
      END
$ENTRY
```

```
      DIMENSION RMS(1800)
      INDEX=0
      PI=4.*ATAN(1.)
      Z=6.
      EC=.84
      Q IS THE VALUE OF LEMDA
      Q=1./SIN(PI/Z)
      CL IS THE CENTRAL DISTANCE BETWEEN THE GENEVA WHEEL AND THE CRANK
      CL=12.
      RC IS THE RADIUS OF THE CRANK
      RC=CL*SIN(PI/Z)
      FD=CL*CL-RC*RC
      GD=SQRT(FD)
      RL=PI/Z
      (XP,YP) ARE THE COORDINATES OF THE POINT  OF ENGAGEMENT
      XP=GD*COS(RL)
      YP=GD*SIN(RL)
      GL=TAN(PI/Z)
      GLP IS THE GRADIENT OF THE LINE CD
      GLP=-1./GL
      YE=3.C0
      XE=5.196152
      C IS THE X- COORDINATE OF THE POINT C1
      C=18.8
 11   C=C+.2
      D=GLP*(C-XP)+YP
      R=(XP-C)**2+(YP-D)**2
;     XB IS THE X- COORDINATE OF THE POINT M
      XB=7.2
 12   RMS1=0.
      RMS2=0.
      CON=XP
      I=1
 13   V=I-1
      W=.1*V
      X=CON-W
      TA=R-(X-C)**2
      IF(TA.LT.0.) GO TO 724
      WAR=SQRT(TA)
      Y=WAR+D
      DY=(-X+C)/(Y-D)
      DDY=-(1.+DY*DY)/(Y-D)
      SZ=(X*X+Y*Y)/(RC*RC)
      CA=(-SZ+Q*Q+1.)/(2.*Q)
      IF(ABS(CA).GT.1.) GO TO 724
C     SIE IS THE ANGULAR POSITION OF THE CRANK
      SIE=ARCOS(CA)
      SIE=-SIE
      SIED=180.*SIE/PI
      B=-Q*SIN(PI/Z)+SIN(PI/Z+SIE)
      A=Q*COS(PI/Z)-COS(PI/Z+SIE)
      ZOM=B/A
      TOM=B*B/A
      ANG=(ZOM*X+Y)/(RC*(A+TOM))
      IF(ABS(ANG).GT.1.) GO TO 724
C     FIE IS THE ANGULAR POSITIN OF THE GENEVA WHEEL
```

```
      FIE=ARSIN(ANG)
      FIED=180.*FIE/PI
      PRI=PI/Z+SIE+FIE
      DSF=((DY*SIN(PRI)+COS(PRI))/(X+Y*DY))*RC
      DXS=-DSF*Y+RC*SIN(PRI)
      DYS=DSF*X-RC*COS(PRI)
      E=(DXS*DXS+DYS*DYS)*DSF
      F=(SIN(PRI)*DXS-COS(PRI)*DYS)*(1.+DSF)*RC
C     DDSF IS THE SECOND TRANSMISSION FUNCTION
      DDSF=(DDY*DXS**3-E-F)/(X*DXS+Y*DYS)
      ZEB=(PI*(Z-2.))/Z
      S=SIE/ZEB
      RA=(2.*EC*EC)/(PI*PI)+(1.-EC)*(1.+2.*EC)/12.
      A=1./RA
      IF(ABS(S)-EC/2.)45,49,49
C     ACCN IS THE DESIRED SECOND TRANSMISSION FUNCTION
 49   ACCN=(4.*Z*A*(S+.5))/(PI*(1.-EC)*(Z-2.)**2)
      DIFN=(ACCN-DDSF)**2
      RMS1=RMS1+DIFN
      PRINT75,X,Y,SIED,FIED,ACCN,DDSF
 75   FORMAT(10X,6F16.4)
      GO TO 46
 45   ACCN=-(2.*Z*A*SIN(PI*S/EC))/(PI*(Z-2.)**2)
      DIFN=(ACCN-DDSF)**2
      RMS1=RMS1+DIFN
      PRINT75,X,Y,SIED,FIED,ACCN,DDSF
 46   IF((X-XB).LT.0.) GO TO 30
      I=I+1
      GO TO 13
 30    XF=XB
 32   YF=SQRT(R-(XF-C)**2)+D
      GRD=(D-YF)/(C-XF)
      AC=(XF*D-YF*C)/(XF-C)
      CXB=(XF+XE)/2.
      CYB=(YF+YE)/2.
      SLOP=(YE-YF)/(XE-XF)
      PSLOP=-1./SLOP
      CP=CYB-CXB*PSLOP
      XL=(AC-CP)/(PSLOP-GRD)
      YL=PSLOP*XL+CP
      RO=(XL-XF)**2+(YL-YF)**2
      FIX=XF
      N=1
 16   VT=N-1
      WT=.1*VT
      X=FIX-WT
      ST=SQRT(RO-(X-XL)**2)
      Y=-ST+YL
      DY=(-X+XL)/(Y-YL)
      DDY=-(1.+DY*DY)/(Y-YL)
      SZ=(X*X+Y*Y)/(RC*RC)
      CA=(-SZ+Q*Q+1.)/(2.*Q)
      IF(ABS(CA).GT.1.) GO TO 20
```

```
      SIE=ARCOS(CA)
      SIE=-SIE
      SIED=180.*SIE/PI
      B=-Q*SIN(PI/Z)+SIN(PI/Z+SIE)
      A=Q*COS(PI/Z)-COS(PI/Z+SIE)
      ZOM=B/A
      TOM=B*B/A
      ANG=(ZOM*X+Y)/(RC*(A+TOM))
      IF(ABS(ANG).GT.1.) GO TO 20
      FIE=ARSIN(ANG)
      FIED=180.*FIE/PI
      PRI=PI/Z+SIE+FIE
      DSF=((DY*SIN(PRI)+COS(PRI))/(X+Y*DY))*RC
      DXS=-DSF*Y+RC*SIN(PRI)
      DYS=DSF*X-RC*COS(PRI)
      E=(DXS*DXS+DYS*DYS)*DSF
      F=(SIN(PRI)*DXS-COS(PRI)*DYS)*(1.+DSF)*RC
      DDSF=(DDY*DXS**3-E-F)/(X*DXS+Y*DYS)
      ZEB=(PI*(Z-2.))/Z
      S=SIE/ZEB
      RA=(2.*EC*EC)/(PI*PI)+(1.-EC)*(1.+2.*EC)/12.
      A=1./RA
      IF(ABS(S)-EC/2.) 61,62,62
 61   ACCN=-(2.*Z*A*SIN(PI*S/EC))/(PI*(Z-2.)**2)
      DIFO=(ACCN-DDSF)**2
      RMS2=RMS2+DIFO
      PRINT75,X,Y,SIED,FIED,ACCN,DDSF
      GO TO 106
 62   ACCN=(4.*Z*A*(S+.5))/(PI*(1.-EC)*(Z-2.)**2)
      DIFO=(ACCN-DDSF)**2
      RMS2=RMS2+DIFO
      PRINT75,X,Y,SIED,FIED,ACCN,DDSF
 106  IF(X-XE)40,40,19
 19   N=N+1
      GO TO 16
 20   N=N-1
 40   INDEX=INDEX+1
      RMS(INDEX)=SQRT(RMS1+RMS2)/(FLOAT(I)+FLOAT(N))
      IF(INDEX/2*2.NE.INDEX)PRINT72,INDEX,RMS(INDEX),C,XB
 72   FORMAT(10X,I10,3F16.4)
      XB=XB+.2
      IF(XB.GT.8.6) GO TO 59
      GO TO 12
 724  XB=XB+.2
      IF(XB.GT.8.6) GO TO 59
      GO TO 12
 59   IF(C.LT.20.) GO TO 11
      STOP
      END
$ENTRY
```

REFERENCES

1. Paul H. Winter - "Calculating Acceleration in Geneva Drive", Machine Design, Vol. 21, No. 2, Feb. 1949, p. 130-131.

2. Otto Lichtwitz - "Mechanism for intermittent motion - Part I", Machine Design, Dec. 1951, p. 134-148.

3. Ray C. Johnson - "How to design Geneva mechanism to minimise contact stress and torsional vibration", Machine Design, Vol. 28, No. 6, March 22, 1956, p. 107-11.

4. K.H. Hunt, N. Flink and J. Nayar - "Linkage Geneva mechanism", Proceedings of the Institution of Mechanical Engineers, Vol. 174, p. 643, 1960.

5. K. Weiss and R.G. Fenton - "Geneva wheel Inertia", Machine Design, April 15, 1965, p.181-182.

6. E.A. Dijksman - "Jerk free Geneva wheel Driving", Journal of Mechanism, Vol. I, 1966, p. 235-283.

7. C.E. Hasty and J.F. Potts - "Analysis and Synthesis procedures of Geneva mechanism design", IBM J. Research & Development, Vol.10, No. 3, May 1966, p. 186-197.